(11) and (12) into (14) and (15) provides the following design equations for the interdigitated directional coupler:

$$Y_o^2 = \left[\frac{k}{2} Y_{11} + \left(\frac{k}{2} - 1\right) \frac{Y_{12}^2}{Y_{11}}\right]^2 - [(k-1)Y_{12}]^2 \tag{18}$$

$$\frac{P_B}{P_{\text{in}}} = \left[\frac{2(k-1)Y_{11}Y_{12}}{kY_{11}^2 + (k-2)Y_{12}^2}\right]^2. \tag{19}$$

It may be more informative to rewrite (18) and (19) in terms of even- and odd-mode admittances, Y_{oe} and Y_{oo}, of a pair of coupled lines which are identical to any adjacent pair of lines, without the presence of others, in the array. Since data of Y_{oe} and Y_{oo} for various coupled pairs of lines are available [6], the values of Y_{oe} and Y_{oo} can be readily translated into a physical configuration. From the definitions of Y_{oe} and Y_{oo}, it is easy to establish the following relations:

$$Y_{11} = \tfrac{1}{2}(Y_{oo} + Y_{oe}) \tag{20}$$

$$Y_{12} = -\tfrac{1}{2}(Y_{oo} - Y_{oe}). \tag{21}$$

Substituting (20) and (21) into (18) and (19), one obtains

$$Y_o^2 = \frac{[(k-1)Y_{oo}^2 + Y_{oo}Y_{oe}][(k-1)Y_{oe}^2 + Y_{oo}Y_{oe}]}{(Y_{oo} + Y_{oe})^2} \tag{22}$$

$$\frac{P_B}{P_{\text{in}}} = \left[\frac{(k-1)Y_{oo}^2 - (k-1)Y_{oe}^2}{(k-1)Y_{oo}^2 + 2Y_{oo}Y_{oe} + (k-1)Y_{oe}^2}\right]^2. \tag{23}$$

When $k = 2$, i.e., for a pair of coupled line, (22) and (23) reduce to the familiar equations:

$$Y_o^2 = Y_{oo}Y_{oe} \tag{24}$$

$$\frac{P_B}{P_{\text{in}}} = \left(\frac{Y_{oo} - Y_{oe}}{Y_{oo} + Y_{oe}}\right)^2. \tag{25}$$

From (22) and (23), various interdigitated couplers using different number of lines for 50-Ω system, i.e., $Y_o = 1/50$, are calculated and listed in Table I.

It should be noted that the effects from bridging connections as well as from junction discontinuities have been neglected in the derivation of the design equations. As a result, there is a practical limitation on the number of lines to be used in the interdigitated coupler. In most applications, however, the choice of $k = 4$ seems to be quite satisfactory.

IV. COMPARISON WITH EMPIRICAL RESULTS

The performance of interdigitated couplers for two different ratios of power coupling, one for 3 dB and another 6.5 dB, has been published in the literature. Waugh and LaCombe [2] reported that a 4-finger 3-dB microstrip coupler can be constructed by using the following physical configuration:

dielectric constant:	$\epsilon_r \simeq 9.5$
substrate thickness:	$h = 0.025$ in
linewidth:	$w = 0.0028$ in
line spacing:	$s = 0.002$ in.

From interpolation in Bryant and Weiss' tables [6], it is estimated that, for this configuration, $Z_{oe} \simeq 170\,\Omega$ and $Z_{oo} \simeq 50\,\Omega$. The theoretical values for a 3-dB coupler with $k = 4$, as calculated in Table I, are $Z_{oe} = 176.2\,\Omega$ and $Z_{oo} = 52.61\,\Omega$, which agree quite well with the empirical results.

A second microstrip coupler, also of 4-finger interdigitated structure for 6.5-dB coupling, was constructed by Miley [3] according to the following data:

TABLE I

No. of Lines k	Power Coupling Ratio (dB)	Y_{oo} (℧)	Z_{oo} (Ω)	Y_{oe} (℧)	Z_{oe} (Ω)
2	3 dB	0.0483	20.71	0.00828	120.7
	6 dB	0.0346	28.87	0.0115	86.60
	10 dB	0.0277	36.04	0.0144	69.37
4	3 dB	0.0190	52.61	0.00568	176.2
	6 dB	0.0147	67.96	0.00702	142.5
	10 dB	0.0131	76.30	0.00845	118.3
6	3 dB	0.0121	82.55	0.00411	243.1
	6 dB	0.00951	105.1	0.00490	204.3
	10 dB	0.00819	122.1	0.00552	181.1

$\epsilon_r \simeq 9.5$
$h = 0.020$ in
$w = 0.003$ in
$s = 0.005$ in.

The even- and odd-mode impedances for this configuration are approximately equal to 135 and 65 Ω, respectively. Using (23), the power coupling is calculated to be about 6.12 dB. Thus, in this case, the calculated coupling and the measured result agree within about 0.2 dB when circuit loss is taken into consideration.

These two examples confirm the close agreement between the theoretical calculation and the measured performance.

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Intermodulation Distortion and Gain Compression in Varactor Frequency Converters

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***Abstract*—Using the nonlinear theory of Gardiner and Ghobrial [1] gain compression in varactor frequency converters is characterized and related to the distortion performance of the device. It is**

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shown that gain compression results from the generation of a current component at the sideband frequency which is in antiphase with the main sideband current component. It is also shown that under certain conditions, high levels of intermodulation distortion can exist with practically zero gain compression.

I. INTRODUCTION

The distortion performance of abrupt junction current pumped varactor frequency converters was successfully analyzed by Gardiner and Ghobrial [1]. The technique that was used employs a nonlinear theory that was capable of relating the distortion performance of the system to the diode parameters as well as the circuit impedance. Using the same approach, it was also possible to prove that A.M. to P.M. conversion does take place in such devices [3], [4]. This technique was extended (with some success) to the case of converters using diffused junction varactors as the mixing device [2].

It is the purpose of the present work to demonstrate that this powerful technique, besides being suitable for predicting the distortion performance of varactor mixers, can also predict gain compression. It will be shown that gain compression can be expressed in terms of the diode parameters and circuit impedances. Such a knowledge is important if gain compression is to be controlled. Furthermore, an attempt to correlate gain compression and intermodulation distortion is made. This resulted in the important conclusion that under certain conditions intermodulation distortion can exist with practically no gain compression, which suggests that gain compression cannot be considered as the mechanism responsible for intermodulation distortion as some workers used to believe [5].

II. ANALYSIS

A. Gain Compression Evaluation

Following the same analysis as given in [1], the sideband current can be written as follows:

$$i_{01} = \frac{-(w_0 + w_1) b V_0 V_1}{\zeta_0 \zeta_1 \zeta_{01} (1 - \gamma)} [1 + B/(1 - B) + B\gamma^* + B^*\gamma + 3B\gamma + \cdots \text{other terms}] \quad (1)$$

where

$$\gamma = \frac{b^2 V_0^2 \exp [j(\phi_1 + \phi_{01})]}{\zeta_0^2 \zeta_1 \zeta_{01}}$$

$$B = \frac{b^2 V_1^2 \exp [j(\phi_0 + \phi_{01})]}{\zeta_0 \zeta_1^2 \zeta_{01}}.$$

All other symbols carry the same meaning as in [1] and [4].

Now, if $V_1 \ll V_0$ (i.e., input signal level is much smaller than that of pump), then the first term in braces will dominate and the above expression reduces to

$$i_{01} = \frac{-(w_0 + w_1) b V_0 V_1}{\zeta_0 \zeta_1 \zeta_{01} (1 - \gamma)}$$

which result is equivalent to that of the linear time varying analysis as was shown in [1] and [4].

From (1) the gain of the converter, in decibels, can be obtained as

$$\text{gain (dB)} = 20 \log_{10} \left| \frac{b V_0 (w_0 + w_1) Z_{01}}{\zeta_0 \zeta_1 \zeta_{01} (1 - \gamma)} \cdot \{1 + B/(1 - B) + 2 |B| |\gamma| \cos\theta + 3B\gamma\} \right| \quad (2)$$

where θ is the angle between the two phasors B and γ (i.e., $\theta = \phi_0 - \phi_1$) and Z_{01} is the impedance Z_n at $(w_0 + w_1)$.

In the small signal case, only the first term need be considered and the gain expression is rendered independent of V_1 which is expected. On the other hand, if the signal level is high, i.e., if V_1 is comparable with V_0, then all four terms should be taken into account. Now, remembering that the quantities B and γ are phasor quantities, it is seen that under certain conditions the last three terms can result in a decrease in the maximum possible gain; i.e., gain compression.

It can be readily proved that the angle θ cannot exceed $\pi/2$ rad, thus the term $2|B||\gamma| \cos\theta$ is always positive and cannot be responsible for gain compression. However, the phase angle associated with the phasor sum of the phasors $B/(1 - B)$ and $3B\gamma$ can assume values greater than $\pi/2$, thus leading to gain compression.

Remembering that $B \ll 1$, we can write

$$B/(1 - B) + 3B\gamma \approx B(1 + 3\gamma).$$

It is obvious that maximum gain compression takes place when the angle associated with the above quantity is 180°. Now if this angle is denoted by μ, then the condition for maximum gain compression can be written symbolically as follows

$$\mu = (\phi_0 + \phi_{01}) + \tan^{-1}\left[\frac{3 \sin (\phi_0 + \phi_{01})}{1 + 3 \cos (\phi_1 + \phi_{01})}\right] = 180°.$$

If this condition is satisfied, then gain compression can be expressed in decibels as follows:

$$\text{gain compression (dB)} = 20 \log_{10} \cdot [1 + 2|B||\gamma| \cos\theta - |B(1 + 3\gamma)|]. \quad (3)$$

It is interesting to note that if μ is 0 or $2n\pi$ rad, then an increase in gain will result.

B. The Relation Between Intermodulation and Gain Compression

Using the definition of intermodulation distortion as given in [1], the following result can be proved to be true:

$$\text{intermodulation distortion} = 20 \log_{10} \cdot |(V_2/V_1) B[1 + 2\gamma + 2\gamma^* \cos\theta \exp (j\theta)]|. \quad (4)$$

Considering (3) and (4) it is seen that both expressions for gain compression and intermodulation distortion are functions of at least four variables and therefore no simple unique relation between these quantities can be derived. It will therefore be wrong to assume that intermodulation distortion is primarily caused by gain compression. In fact, intermodulation distortion can exist even in the absence of gain compression if $\mu = 2n\pi$, as was previously mentioned.

III. COMPUTED AND MEASURED RESULTS

Description and circuits of the experimental up-converter and measuring setup will be found in [1] and [4]. Figs. 1 and 2 show plots of measured gain and intermodulation distortion. Theoretical curves are also included for comparison. The latter were obtained by evaluating (2) and (4) for different V_1. It is worth noting that in the distortion measurements V_2 had the same numerical value as V_1. From these graphs it is seen that the agreement between theory and practice is good. The discrepancy of 1 dB between measured and predicted gain can be attributed to losses in the tuned circuits and the diode. These effects were not taken into account in the present theory.

In the course of measurements it was noticed that if the pump voltage is increased and the system is tuned for maximum gain (hot tuning), then higher gain can be achieved with better distortion performance. However, if the pump voltage exceeds some critical value, then the system becomes unstable with a very high level

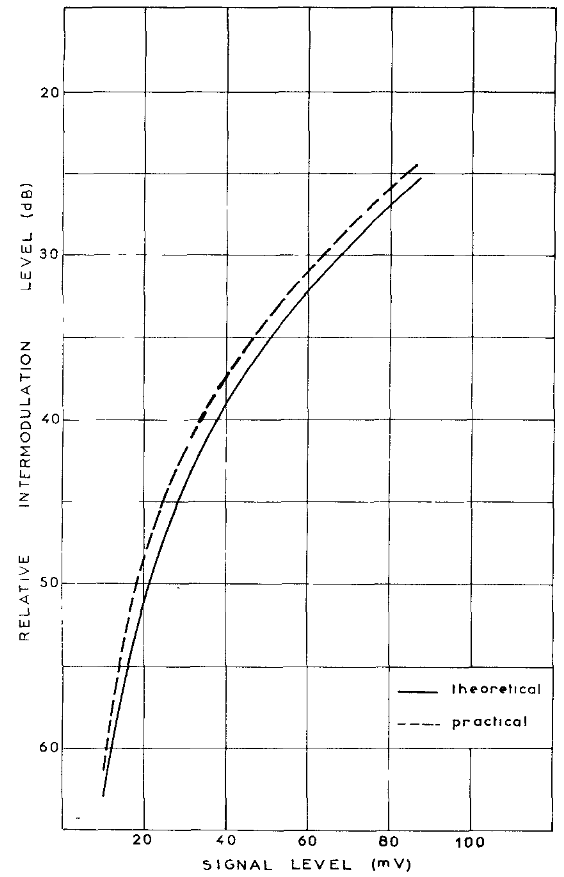

Fig. 1. Distortion performance of up-converter. $B = 1.4 V_1^2 \exp(j140)$; $\gamma = 0.54 \exp(j59.3)$.

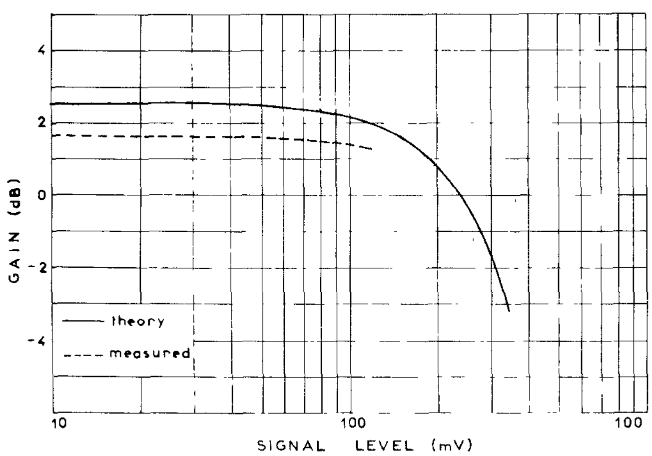

Fig. 2. Variation of gain with signal level.

of distortion. Improvements of 2 dB in gain and 3 dB in distortion level were achieved by increasing the pump voltage from 1 V to 2.2 V.

IV. CONCLUSIONS

It can be concluded that although gain compression and intermodulation distortion are caused by the nonlinearity of the mixing device, one cannot attribute intermodulation distortion to gain compression. This conclusion was confirmed by measurements.

It can also be concluded that gain compression is caused by the generation of a current component at the sideband frequency which is in antiphase with the main sideband current.

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Slant Dielectric Interface Discontinuity in a Waveguide

S. C. KASHYAP

***Abstract*—The reflection and transmission of electromagnetic waves by a slant interface between two dielectric media is investigated. By using suitable Green's functions and a geometrical optics approximation for the field on the dielectric interface, expressions for the transmitted and reflected fields are derived. The approximate results obtained in this manner are compared with the available numerical data and are shown to be fairly accurate for a number of cases of interest.**

INTRODUCTION

Recently, considerable attention has been devoted to the reflection and transmission properties of a slant interface between two dielectric media in a rectangular waveguide. Chow and Wu [1] introduced a moment method with mixed basis functions and applied it to this problem. De Jong and Offringa [2] used suitable waveguide Green's functions to obtain integral representations for the reflected, transmitted, and the unknown field distributions on the interface. Both investigations employed numerical methods for integration. It is the purpose of this short paper to present a much simpler approach by using waveguide Green's functions and a geometrical optics approximation for the field on the interface. In order to obtain this field, the incident field is first divided into two TEM plane waves propagating at angles $\pm\theta_i$ with respect to the waveguide axis. Reflected and transmitted amplitudes of these plane waves, as well as their new directions in each medium, are determined by well-known Fresnel formulas. These plane waves are then utilized to find the fields and their normal derivatives on the interface. Once the fields and their normal derivatives are known, the reflection and transmission coefficients can be determined by using Green's theorem and appropriate Green's functions. The approximate results obtained in this manner are compared with the available numerical data.

THEORY

Consider the slant dielectric discontinuity of Fig. 1 with a TE_{10} mode incident from region I on the interface. The normalized incident electric field is given by

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